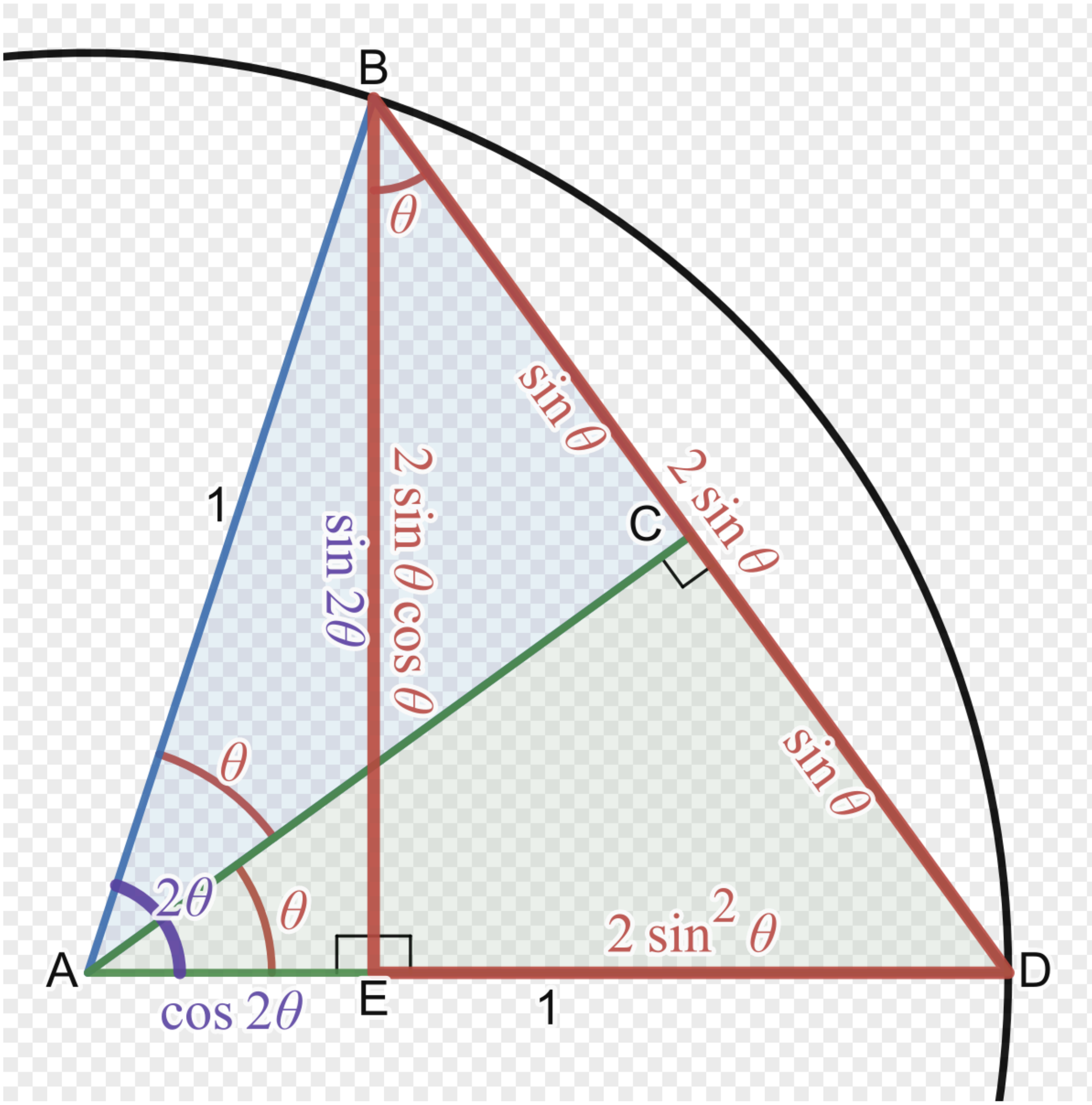
Pre-Calculus



[1]

[1] https://commons.wikimedia.org/wiki/File:Diagram_showing_how_to_derive_the_power_reducing_formula_for_sine.svg

Calculus is the study of continuous change.

We start Calculus with a definition, which you don't need to know right now. Enjoy the preview.

<u>Definition:</u>

The limit of f (x) , as "x approaches p" , exists and equals L ,

If there exist a δ , such that: $0 < | x - p | < \delta$

implies $0 < | f(x) - L | < \varepsilon$, for all possible ε.

We write:

$$\lim_{x \to p} f(x) = L,$$

x, p, L are all real numbers in the real number system by the way.[2]

This leads to the definition of the **derivative**:

$$L = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

f ‘ (x) equals L.[3]

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[2] https://en.wikipedia.org/wiki/Limit_of_a_function#(%CE%B5,_%CE%B4)-definition_of_limit
[3] https://en.wikipedia.org/wiki/Derivative

PreCalculus course work involves the following:

10) Conic Sections

11) Probability and Combinatorics

12) Series

13) Limits and Continuity

Composite Functions:

$$(g \circ f)(x) = g(f(x)).$$ [4]

For functions f(x) and g(x), "g of f(x)" as we would say, is expressed with the above equation and notation.

Example:

Let $f(x) = 2x$, and let $g(x) = x^2$ to begin with.

$(g \text{ o } f)(x) = g((2x))$.

$= ((2x)^2)$

[4] https://en.wikipedia.org/wiki/Function_composition

Test points:

x	$2x$	$4x^2$
1	2	4
2	4	16
3	6	36

So if we plug in 1 for x into f(x) , our output is 2. Then 2 is passed through g(x) and squared to equal 4.

Polynomial and Rational Functions

$$y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

n is a non-negative integer that defines the degree of the polynomial. n can be 0, 1, 2, etc.

An example of a polynomial function is

$$y = x^2 + 4x + 4 .$$

The degree of the polynomial is 2.

$a_2 = 1$.
$a_1 = 4$.
$a_0 = 4$.

The a's are constant coefficients.

If p(x) and q(x) are also polynomial functions,

$$Z(x) = p(x) / q(x) \quad .$$

is also a rational function. q(x) is never equal to 0.

Exponential Functions and Logarithmic Functions

$$b^n = \underbrace{b \times b \times \cdots \times b \times b}_{n \text{ times}}.$$ [5]

b is known as the base and n is called the exponent.

$$y = b^x .$$

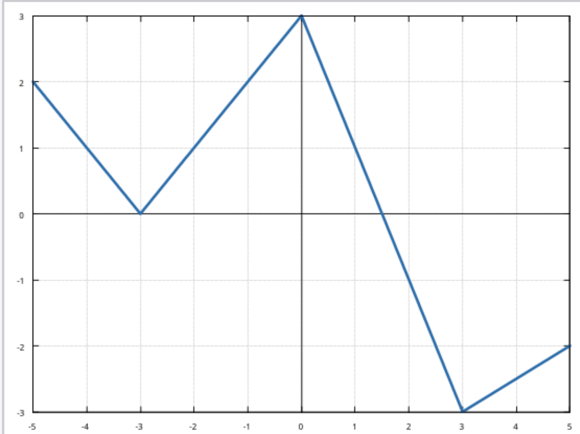
is an exponential function with constant base *b*.

$$log_b\, y = x.$$

The logarithmic function above is the inverse function of the general exponential function. The output of a logarithmic function is an exponent.

[5] https://en.wikipedia.org/wiki/Exponential_function

Piecewise Linear Functions



Plot of the piecewise linear function

$$f(x) = \begin{cases} -3 - x & \text{if} \quad x \leq -3 \\ x + 3 & \text{if} \quad -3 \leq x \leq 0 \\ 3 - 2x & \text{if} \quad 0 \leq x \leq 3 \\ 0.5x - 4.5 & \text{if} \quad 3 \leq x \end{cases}$$

[6]

Piecewise functions are what they sound like. For a given piece of an interval, there is an output that doesn't behave like the other parts of the graph.

For example in the above graph when x is less than or equal to 3, we have a downward sloping line that would intercept the y axis at (0 , -3) . Its slope is -1.

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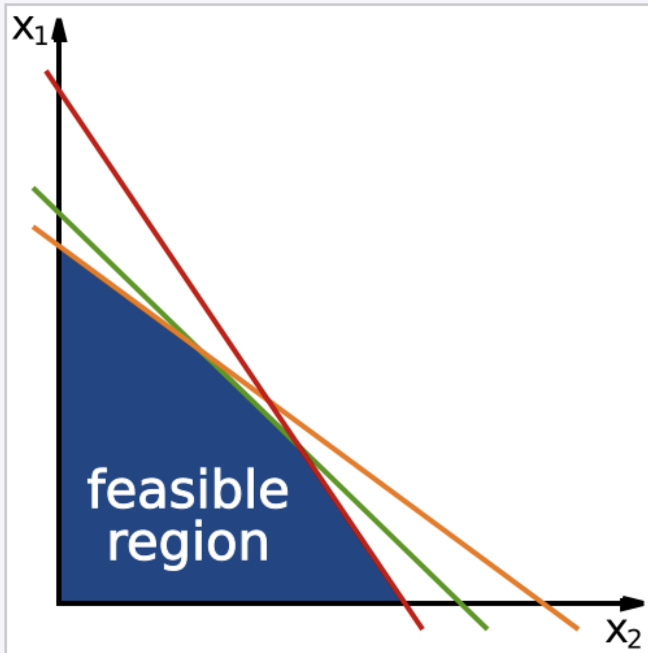
[6] https://en.wikipedia.org/wiki/Piecewise_function

How does the graph behave when our inputs for x are between -3 and 0, including the endpoints?

Inequalties, Linear and Quadratic

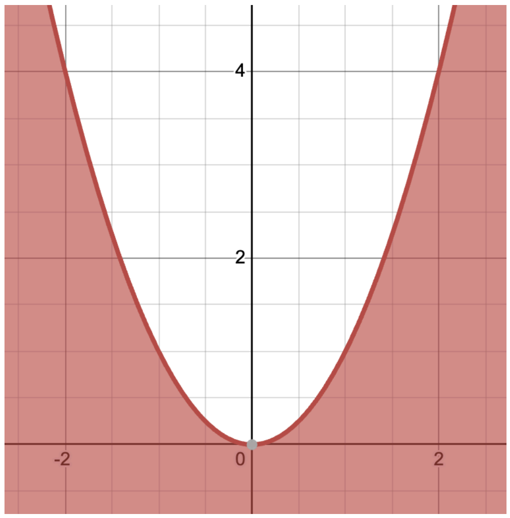


The feasible regions of linear programming are defined by a set of inequalities.[7]

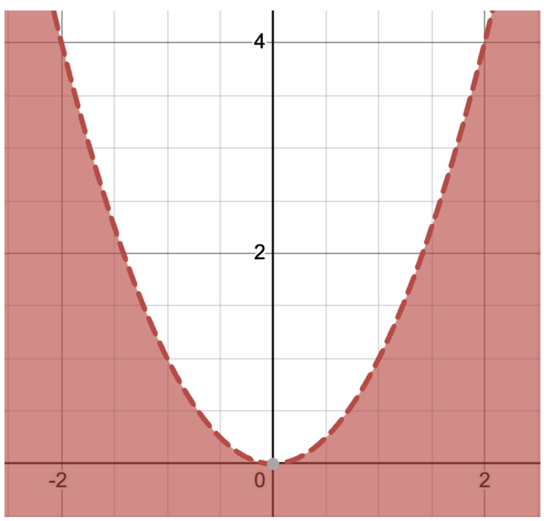
The above graph is just an intuitive picture, we will work with more specific examples.

[7] https://en.wikipedia.org/wiki/Inequality_(mathematics)

<u>Example:</u>

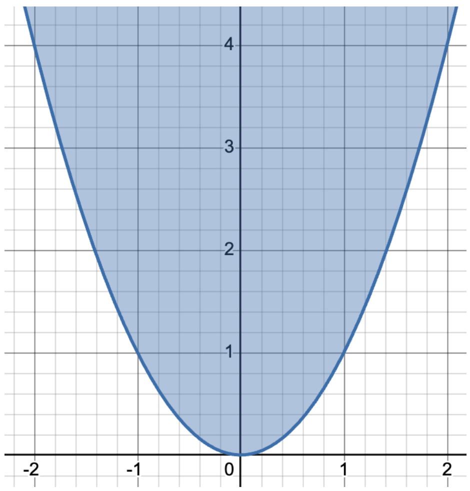


The above graph is of $y \leq x^2$. Note that we shade <u>below</u>, and the line itself is **solid**.

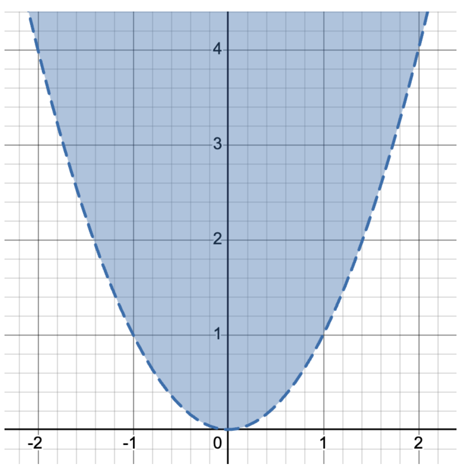


This second graph is $y < x^2$. We again shade <u>below</u>, but the line itself is *dotted*.

Example:

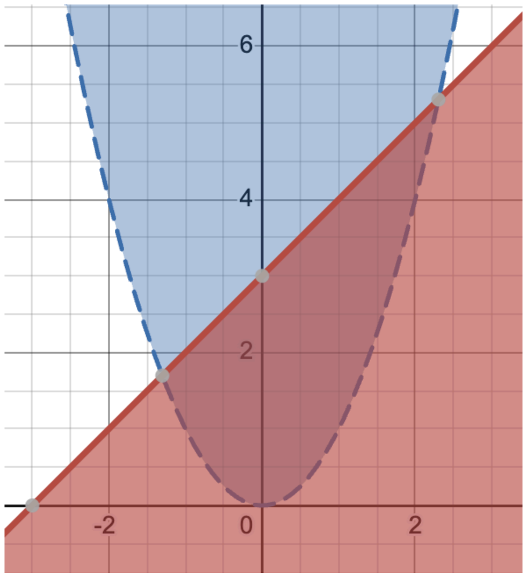


The above graph is of $y \geq x^2$. Note that we shade above, and the line itself is **solid**.

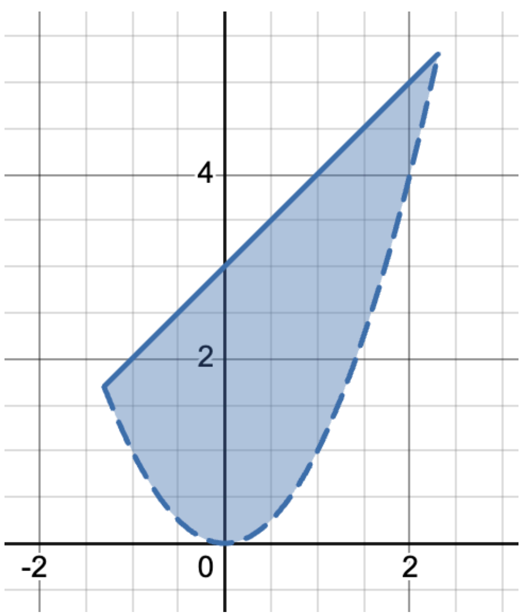


This next graph above is $y > x^2$. Note that we shade above, and the line itself is *dotted*.

Example:



Here we have $y > x^2$ and $y \leq x + 3$.



You would shade like this so to show the solution set of points. Notice how one of the lines is **solid**.

Rational Root Theorem

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 = 0$$

Essentially the Rational Root Theorem states that the above rational polynomial of non-negative integer degree n has at most n ***roots***.

Roots are the x-coordinates where the graph crosses the x-axis, the y-coordinate equals 0, and the given polynomial or function equates to 0 for a given x input.

For example:

$x^2 + 4x + 3$ equals 0 ,

when x = -1, or x = -3 .

x = -1 or x = -3 are the roots of the above polynomial.

Joint Variation

Example:

$$\varphi\ (x,\ t) = \mathrm{k}xy\ .$$

where x and y are independent variables and k is a constant coefficient.